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THE STABILITY OF A FLUID LAYER HEATED UNIFORMLY FROM BELOW

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Abstract — The onset of convective motion in a horizontal fluid layer under constant flux heating is analyzed by the modified frozen-time analysis confining temperature disturbances within a thermal penetration distance. For this purpose a new mathematical method is introduced in detail. It is found that the present stability analysis predicts the most reliable critical conditions based on the comparisons with the published experiments and theoretical models. At $\tau_c < 0.01$, the stability conditions lead to $Ra_c = 14.1\ \tau_c^{-2}$ and $a_c = 0.248\ Ra_c^{1/4}$ and the period required for the growth of disturbances to a detectable size is inversely proportional to $Ra_c^{1/2}$.

INTRODUCTION

When a horizontal layer of intially motionless fluid is heated from below, buoyancy-driven convection occurs in the range exceeding a certain adverse temperature gradient. The increase of heat transport induced by such free convection is of practical importance in connection with energy transmission systems, e.g., heat exchangers. Since Bénard [1] reported the systematic experimental results associated with thermal convection, numerous contributions have been made to this field. When heat is introduced slowly to the lower boundary of the system, it is well known that the linearized theory of Rayleigh [2] produces the stability conditions consistent with the experimental results. However, the stability analysis in the system with a nonlinear base temperature profile is not so well established as the above case of linear base temperature profiles.

In the present study the extended problem of Rayleigh-Bénard instabilities, in which the base temperature is time-dependent and vertically nonlinear, is examined using the linear stability theory. The system of particular interest is a fluid layer confined between two horizontal rigid plates and heated uniformly from below with constant heat flux. In the present system the time required for the onset of free convection at a given heating rate becomes a dominant question. This kind of eigenvalue problems have been investigated thoroughly since Morton [3] employed the marginal stability analysis freezing the instantaneous time at the onset of convective motion. Lick [4] and Currie [5] extended this frozen-time model, approximating the nonlinear base temperature profile by two linear segments. As another version of stability, Foster [6] developed the initial value technique to describe the time development of disturbances in the system. The major subsequent work of this amplification theory has been conducted by Foster [7] and Gresho and Sani [8]. They tried to make a prediction of the time of onset of manifest convection, considering some amplification factor. But this amplification theory involves the inherent difficulty in determining the initial conditions and the magnitude of first observable motion.

Recently Davis and Choi [9] suggested a modification of frozen-time analysis that for a large Prandtl number temperature disturbances are confined to some effective thermal depth, i.e., a penetration distance. In plane Couette flow of water they showed that this modified stability analysis produces the best agreement with experimental results. Kim et al. [10] applied the modified concept to the uniformly heated fluid layer. Using the Galerkin method, they reported that the modified analysis produces the most reasonable stability criteria in comparison with the experiments of Nielsen and Sabersky [11]. The purpose of this study is to critically reexamine the instability of the above case, using a new

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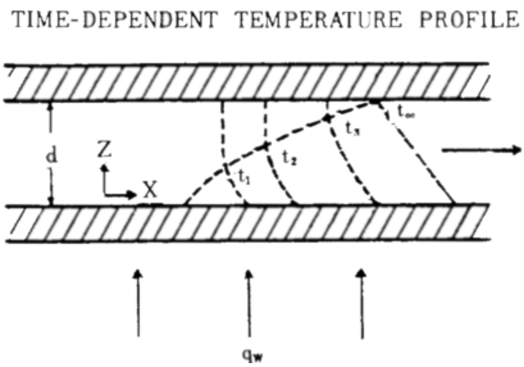

Fig. 1. Sketch illustrating conduction layer.

semi-analytical approach and to improve drastically the means of predicting the onset of free convection.

FORMULATION OF GOVERNING EQUATIONS

Consider an intially quiescent layer of Newtonian fluid between two rigid plates, as illustrated in Figure 1. The base temperature profiles caused by pure conduction will evolve, as indicated in the figure. For a given heat flux q_w, buoyancy-driven convection will set in at a certain critical time. The problem is to find this critical time when the fluid layer becomes unstable with respect to a given heat flux.

In the present system the conduction equation becomes, in a dimensionless form,

$$\frac{\partial\theta_0}{\partial\tau}=\frac{\partial^2\theta_0}{\partial z^2} \qquad (1)$$

with the initial and boundary conditions as

$$\theta_0=0 \quad \text{at} \quad \tau=0$$
$$\theta_0=0 \quad \text{at} \quad z=1 \qquad (2)$$
$$\frac{\partial\theta_0}{\partial z}=-1 \quad \text{at} \quad z=0$$

The above dimensionless variables are defined by $(x,y,z)=\frac{1}{d}(X,Y,Z)$, $(u,v,w)=\frac{d}{\alpha}(U,V,W)$, $\theta=\frac{k(T-T_i)}{q_w\, d}$, and $\tau=\frac{t\alpha}{d^2}$, where α denotes the termal diffusivity, k the thermal conductivity and T_i the initial temperature. This conduction equation is easily solved, using the method of separation of variables (Graeta procedure) as follows:

$$\theta_0=1-z-\sum_{n=1}^{\infty}\frac{2}{\lambda_n^2}e^{-\lambda_n^2\tau}\cos\lambda_n z \qquad (3)$$

where $\lambda_n=(n-\frac{1}{2})\pi$. However, at small times of $\tau\leqq 0.01$ the following Leveque solution is more convenient:

$$\theta_0=\sqrt{\frac{4\tau}{\pi}}\exp\left(-\frac{z}{4\tau}\right)-\sqrt{z}\left\{1-\mathrm{erf}\left(\sqrt{\frac{z}{4\tau}}\right)\right\} \qquad (4)$$

The dimensionless perturbation equations for disturbances under the Boussinesq approximation are derived as usual (for example, see Davis and Choi 1977):

$$\left[\frac{1}{Pr}\frac{\partial}{\partial\tau}-\left(\frac{\partial^2}{\partial z^2}-a^2\right)\right]\left(\frac{\partial^2}{\partial z^2}-a^2\right)\omega^*+R_a\, a^2\,\theta^*=0 \qquad (5)$$

$$\left[\frac{\partial}{\partial\tau}-\left(\frac{\partial^2}{\partial z^2}-a^2\right)\right]\theta^*+\omega^*\frac{\partial\theta_0}{\partial z}=0 \qquad (6)$$

where ω^* and θ^* represent the x-and y-independent amplitudes of disturbances. These functions are derived from the assumptions that any disturbance in the infinite horizontal xy-plane may be expressed in terms of a periodic horizontal wave number a. The important parameters to describe the system, the Prantl number Pr and the Rayleigh number based on heat flux Ra, are defined by $Pr=\frac{\nu}{\alpha}$ and

$$Ra=\frac{g\beta\, q_w\, d^4}{\alpha\, k\, \nu},$$

were ν denotes the kinematic viscosity, g the gravity acceleration and β the thermal expansivity. Additionally, the Nusselt number characterizing thermal enhancement is defined by $Nu=1/\theta_w$, in accordance with that of Nielsen and Sabersky [11], where θ_w is the wall temperature at the heated surface. Using the above equations, the critical conditions (τ_c vs. Ra_c and Ra_c vs. a_c) marking the onset of convective motion must be found subject to the boundary conditions:

$$\omega^*=\frac{\partial\omega^*}{\partial z}=\frac{\partial\theta^*}{\partial z}=0 \quad \text{at} \quad z=0 \qquad (7)$$

$$\omega^*=\frac{\partial\omega^*}{\partial z}=\theta^*=0 \quad \text{at} \quad z=1 \qquad (8)$$

For the same system Nielsen and Sabersky [11] conducted the experiments with silicone oils of Pr = 45~4,770. Their results of Nu vs. τ with respect to Ra are summarized in Figure 2. The exact solution (3) of pure conduction represented by the solid curve is found to be consistent with the experimental results. With the increase of Ra the deviation of Nu from the solid curve starts at a smaller time. Nielsen and Sabersky considered the critical time as the time at which motion was first observed on the shadowgraph. In the figure their data points are represented by the dashed curve. It is easily understood that disturbances caused by the onset of motion require some growth period until they are detectable. Since the detection of motion is dependent upon the sensitivity of a measuring apparatus, such consideration of a critical time as the time of manifest convection that has been claimed by Foster et al. lacks uniqueness. Therefore, in the present study we regard

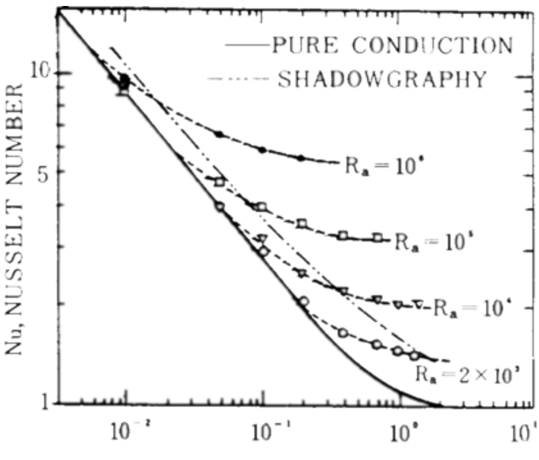

Fig. 2. Heat transfer characteristics based on experiments of Nielsen and Sabersky[11].

the critical time as the time at which Nu deviates from the pure conduction state.

Even though the conduction solution is easily obtained, the nonlinear term $\frac{\partial \theta_0}{\partial z}$ in the equation (6) brings the mathematical difficulty in performing the stability analysis. Thus, a modification of conduction solution is made:

$$\theta_0 = \theta_{0,w}\left(1-\frac{z}{\delta}\right)^{\delta/\theta_{0,w}} \quad \text{for} \quad z \le \delta \tag{9}$$

$$\theta_0 = 0 \quad \text{for} \quad z \ge \delta \tag{10}$$

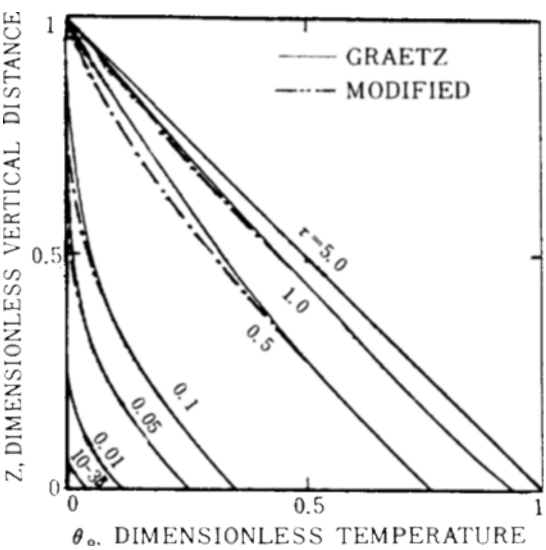

Fig. 3. Comparison of base temperature profiles between the exact Graetz solution and its modification.

where δ is taken as a vertical distance from the bottom surface at which $\theta_0 = 0.01\ \theta_{0,w}$. This distance is a so-called thermal penetration distance. The values of δ and $\theta_{0,w}$ are obtained from the equation (3) or (4). The base temperature distributions predicted by the above modification are compared with the exact ones in Figure 3. The agreement between two cases is found to be good. It is noted that the present approximation is a more accurate one than the two linear segment approximation of Currie [5]. As will be illustrated in the next section, the equations (9) and (10) make it possible to generate the critical conditions analytically with a good accuracy.

STABILITY ANALYSIS

As the perturbation equations are still much complicated, two additional assumptions of (a) the onset of stationary motion and (b) Pr →∞ are made. The first assmption makes the governing equations time-independent and the second one makes it possible to apply the modified frozen-time analysis to the present system without the loss of physical implication of Pr.

Kim et al. [10] conducted the stability analysis for the present system, using the following trial functions based on the work of Gresho and Sani [8]:

$$\omega^* = \sum_{j=1}^{\infty} A_j \sin \pi z \ \sin j\pi z \tag{11}$$

$$\theta^* = \sum_{j=1}^{\infty} B_j \ \cos\left(j-\frac{1}{2}\right)\pi z \tag{12}$$

Their results from the Galerkin scheme show that the conventional frozen-time analysis is valid only for the case of a nearly linear base temperature profile, while the Choi et al. 's modified one agrees well with the experiments of Nielsen and Sabersky [11] in the entire range. Since the Galerkin method is an approximate approach, the degree of correctness becomes doubtful owing to its poor convergence at small times. Therefore, the more systematic stability analysis involving the modified concept will be conducted analytically, using the equations (6) and (7).

With the modified frozen-time analysis confining temperature disturbances within a thermal penetration distance the equations (5) and (6) are transformed to

$$[(D^2-\delta^2a^2)^2+Ra\ a^2\delta^4(1-\zeta)^{\delta/\theta_{0,w}-1}]\omega_A^* = 0$$
$$\text{for} \quad 0 \le \zeta \le 1 \tag{13}$$

$$(D^2-\delta^2a^2)^2\omega_B^* = 0 \quad \text{or} \quad \theta_B^* = 0$$
$$\text{for} \quad 1 \le \zeta \le 1/\delta \tag{14}$$

where $\zeta = z/\delta$ and $D = \frac{d}{d\zeta}$. The subscript A denotes the region inside of a penetration distance and B its outside region. Introducing the interface conditions of Currie [5] at $\zeta = 1$ and rewriting the boundary conditions (7) and

(8), the complete set of boundary conditions become

$$D^n\omega_A^* = D_n\omega_B^* \quad (n=0,\ 1,\ 2,\ 3,\ 4) \quad \text{at } \zeta = 1 \qquad (15)$$

$$\omega_A^* = D\omega_B^* = D(D^2-\delta^2 a^2)^2\omega_A^* = 0 \quad \text{at } \zeta = 0 \qquad (16)$$

$$\omega_B^* = D\omega_B^* = 0 \qquad \text{at } \zeta = 1/\delta \qquad (17)$$

However, as the equation (13) is still complicated, the base temperature gradient is expanded in a Taylor series:

$$(1-\zeta)^{\delta/\theta_{0,w}-1} = 1+\sum_{n=1}^{\infty}(-1)^n \{\prod_{k=1}^{n}(\frac{\delta}{k\,\theta_{0,w}}-1)\}\zeta^n \qquad (18)$$

Now, we can obtain the general solution, using a rapidly converging power series similar to those developed by Sparrow et al. [12] as follows:

$$\omega_A^* = \sum_{i=0}^{5} C_i\, f^{(i)}(\zeta) \qquad (19)$$

$$f^{(i)}(\zeta) = \sum_{n=0}^{\infty} b_n^{(i)}\zeta^n \qquad (20)$$

where C_i(i = 0 to 9) are arbitrary constants. The series coefficients are defined by $b_{-1}^{(i)} = 0$ and $b_n^{(i)} = \delta_{ni}$ (n = 0 to 5), where δ_{ni} indicates the Kronecker delta. For n≥6 the following recursion formula is generated from the equation (10):

$$b_n^{(i)} = \frac{1}{n!}[3\gamma^2(n-2)!\,b_{n-2}^{(i)} - 3\gamma^4(n-4)!\,b_{n-4}^{(i)} + (\gamma^4 - Ra\delta^4)\gamma^2(n-6)!\,b_{n-6}^{(i)} - Ra\delta^4\gamma^2(n-6)!\sum_{j=-1}^{n-7}(-1)^{n-6-j}\{\prod_{k=1}^{n-6-j}(\frac{\delta}{k\,\theta_{0,w}}-1)\}b_j^{(i)}] \qquad (21)$$

where $\gamma = \delta a$. From the boundary condition (16) at ζ = 0 we obtain $C_0 = C_1 = 0$ and $C_5 = \frac{\gamma^2}{10}C_3$.

The general solution of equation (14) is easily obtained as follows:

$$\omega_B^* = (C_6 + C_7\zeta)e^{-\gamma\zeta} + (C_8 + C_9\zeta)e^{\gamma\zeta} \qquad (22)$$

From the above solutions and remaining boundary conditions seven algebraic equations are obtained in terms of seven constants C_n (n = 2,3,4,6,7,8,9). The resulting secular equation characterizing the stability conditions is

$$[\underline{F}(\frac{1}{\delta})\ D\underline{F}(\frac{1}{\delta})\ \underline{G}(1)\ D\underline{G}(1)\ D^2\underline{G}(1)\ D^3\underline{G}(1)\ D^3\underline{G}(1)\ D^4\underline{G}(1)]^T = 0 \qquad (23)$$

where $\underline{F}$ and $\underline{G}$ are column matrices of seven elements as the function of ζ and the superscript T indicates the transpose of a matrix. $\underline{F}$ and $\underline{G}$ are given as follow:

$$\underline{F} = [0\ 0\ 0\ e^{-\gamma\zeta}\ \zeta e^{-\gamma\zeta}\ e^{\gamma\zeta}\ \zeta e^{\gamma\zeta}]^T \qquad (24)$$

$$\underline{G} = [-f^{(2)}(\zeta)\ \ -\{f^{(3)}(\zeta)+\frac{\gamma^2}{10}f^{(5)}(\zeta)\}\ \ -f^{(4)}(\zeta)\ \ e^{-\gamma\zeta}\ \ \zeta e^{-\gamma\zeta}\ \ e^{\gamma\zeta}\ \ \zeta e^{\gamma\zeta}]^T \qquad (25)$$

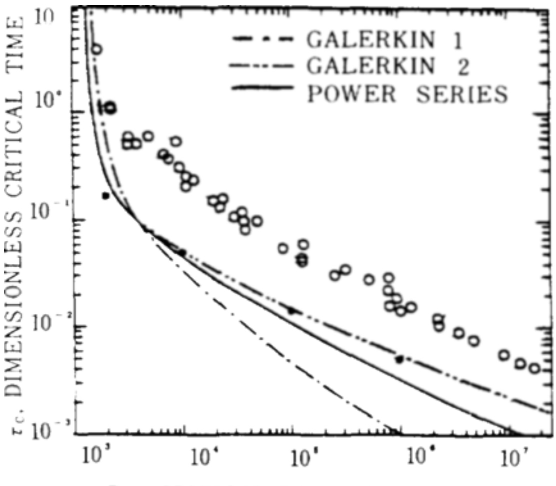

Fig. 4. Comparison of theoretical models with experiments of Nielsen and Sabersky [11]. Galerkin method, the work of Kim et al. [10]; o, detection of motion on the shadowgraph.

The eigenvalues contained in the equation (23) are Ra, a and τ_c. For the given τ_c the neutral stability curve of Ra vs. a must be computed. For that particular time the minimum value of Ra is the critical Rayleigh number Ra_c and its corresponding wave number is the critical wave number a_c.

GENERAL RESULTS AND DISCUSSION

The variations of the Rayleigh number with respect to the critical time marking the onset of free convection are summarized in Figure 4. In the figure a dot indicates the deviation point of Nu from that of pure conduction for a given Ra (see Figure 2) and a circle the detection time of motion observed on the shadowgraph. All these experimental results were obtained by Nielsen and Sabersky [11], using silicone oils of Pr≥45. The curve "Galerkin 1" represents the theoretical results based on the conventional frozen-time analysis, using the Galerkin method, and "Galerkin 2" its modification. This Galerkin procedure using equations (11) and (12) is described in the work of Kim et al. [10], but its convergence is noted to become poorer as τ_c decreases. The present theoretical results using a rapidly converging power series (19) to (21) is found to constitute a reasonable lower bound of experimental data. As mentioned before, the detection of motion requires the growth period of disturbances to a detectable size. It should be observed that at large critical Rayleigh numbers the present results on logarithmic scales is almost parallel to the trajectory of the experimental

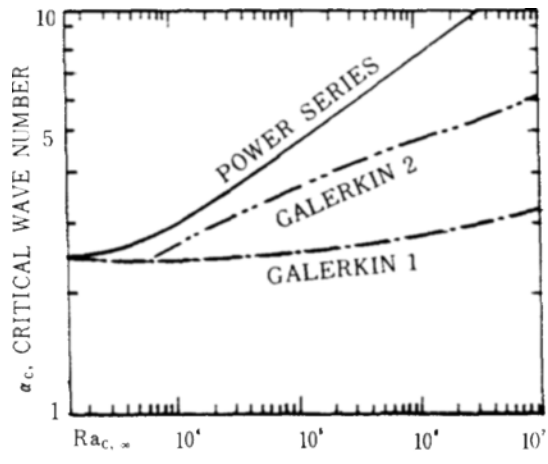

Fig. 5. Critical wave number as a function of Rayleigh number.

points.

The critical wave number as a function of the Rayleigh number is shown in Figure 5. Differences among these theoretical results may be explained from both physical and mathematical viewpoints. Unfortunately no experimental data exist in this connection, but it is believed that the wave number decreases with the decrease of the Prandtl number. Since the present results are based on the limiting case of $Pr \to \infty$, they may provide the upper bound of the critical wave number. Comparing the results of Currie [5] with the present study, his Ra_c is a little lower than those of the presnt study, but in the range of $Ra_c \geq 10^6$ his a_c agrees well with those of "Galerkin 2". In the case of rapid heating the wave number of growing disturbances will be continuously changing. This fact makes it very difficult to determine experimentally the critical wave number marking the onset of motion in the transient system.

With the increase of τ_c the critical conditions lead to $Ra_{c,\infty} = 1295.8$ and $a_{c,\infty} = 2.55$, which are the same values as those in the case of a linear base temperature profile. On the other hand, with the decrease of τ_c the present analysis shows the relations of $Ra_c = 14.1\,\tau_c^{-2}$ and $a_c = 0.248\, Ra_c^{1/4}$. These expressions may be used for the prediction of critical conditions in the range of $\tau_c \leq 0.01$. In this range, the ratio of δ to $\theta_{o,w}$ has the value of 2.84 as shown in Figure 6, and the relation of $\delta = 3.21\,\tau_c^{1/2}$ may be used, considering the solutions (3) and (4).

Applying the condition of $\delta \to 0$ or $\tau_c \to 0$ to the equations (13) and (14), the present system becomes artificially independent of δ and the new eigenvalues of $Ra\delta^4$ and a may be obtained. These limiting values are found to be $Ra_c\delta^4 = 1{,}490$ and $a_c\delta = 1.54$. They are valid only when the Prandtl number is very large. The modified analysis may not be applied to the system with samll Prandtl numbers. Davis and Choi [9] showed that the modified analysis predicts the oneset of free convection very well for water data of $Pr = 6 \sim 8$. Thus, it may be stated that the present analysis provides the most reasonable stability criteria of fluids in the range of $Pr \geq 6$. The study of the effect of Pr on the critical conditions is in progress in this laboratory.

At this stage it may be of interest to convert the present results in terms of the conventional Rayleigh number $R = g\beta \Delta T d^3/(\alpha\nu)$, based on the overall temperature difference. The relationship between R and Ra can be written as $R = \theta_{o,w}$ Ra. In the range of $\tau_c \leq 0.01$ the correlations of $R_c\delta^3 = 526$ and $R_c = 2.19\, Ra_c^{3/4}$ are easily obtained. For a given Ra_c in the range of $10^4 < Ra_c < 10^7$ the latter correlation produces the lower value of R_c than that of Nielsen and Sabersky [11], but produces the higher one than that of Soberman [13]. In their correlation Nielsen and Sabersky considered the critical conditions as the time at which motion is first detected on their shadowgraph. Thus, it seems reasonable that their results are higher than the present ones which are concerned with the instant of beginning disturbances. It has been generally accepted that the experimental results of Soberman are incorrect owing to an underestimate of the overall temperature difference, but they can be used as a lower bound of R_c.

The slope of R_c vs. Ra_c is almost the same as that of Nielsen and Sabersky. Considering the sensitivity of their shadowgraph, it may be presumed that for the same magnitude of disturbances a period of growth of disturbances decreases with the increase of the Rayleigh

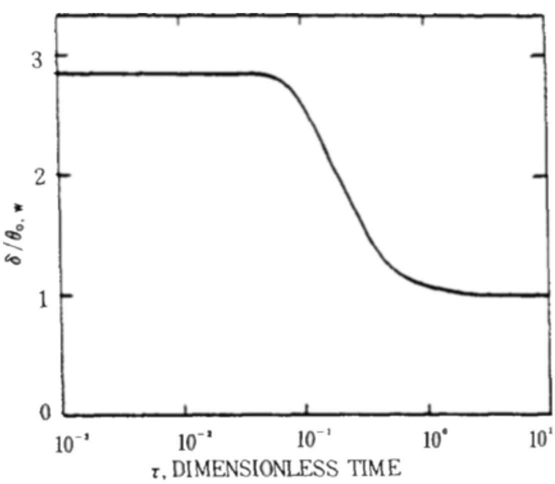

Fig. 6. The ratio of penetration distance to wall temperature as a function of time.

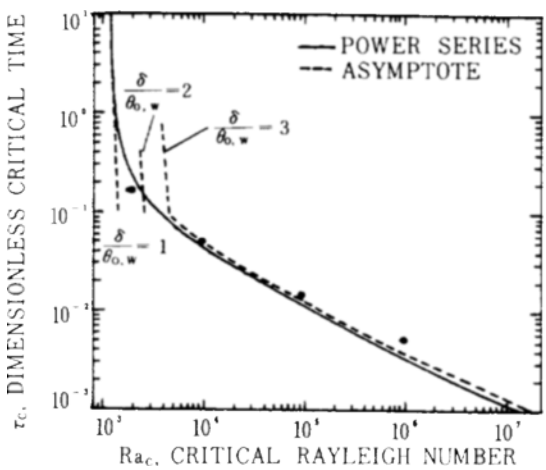

Fig. 7. Comparison of asymptotic analyses with the present analyses.

number. With $Ra_c<10^4$ the growth period based on the present analysis and their detection time is approximated: $\Delta\tau_c = 12.1\ Ra_c^{-1/2}$. In other words disturbances grow rapidly in proportion to $Ra_c^{1/2}$. These trends are qualitatively in agreement with those of Gresho and Sani [8]. Once motion is detected, the linear theory will be no longer applicable.

For the simplification of mathematical procedure in the present system Kim and Choi [14] conducted the asymptotic analysis, choosing the values of $\delta/\theta_{0,w}$ as 1, 2 and 3. In Figure 6 it is shown that at $\tau_c>2$, its value converges to 1 and at $\tau_c<0.05$ it approaches a constant of 2.84. Their asymptotic results are compared with the present one in Figure 7. When $\tau_c<0.1$ and $\tau_c>2$, the agreement between two analyses is excellent, including the critical wave number. Thus, this kind of analysis will provide the preliminary criteria of stability with ease. Also, it is known that the shape of base temperature profiles is important in determining the critical conditions.

In the system of $Pr\to\infty$ the momentum boundary-layer thickness is very large in comparison with the thermal one. In the present analysis this premise makes it possible to confine temperature disturbances within a thermal penetration distance and also causes velocity disturbances to move through the whole depth of a fluid layer. The slow heating removes the dependence of stability conditions on Pr, because both thermal and momentum boundary layers are fully developed to the whole fluid depth upon the onset of motion. But in the case of rapid heating there remains a question about the effect of Pr, which will control the range of propagation of disturbances of temperature and velocity. In this viewpoint the conventional frozen-time analysis assuming that disturbances of both temperature and velocity will occupy the whole fluid depth may correspond to the limiting case of $Pr\to 0$. But considering the driving force of temperature difference causing the onset of motion, it is also a probable guess that Ra_c is a weak function of Pr, but a_c is dependent upon Pr to a certain degree.

Considering all of the present results, the stability criteria are strongly dependent upon both the shape of a base temperature profile and the range of propagation of disturbances. It may be loosely stated that the modified frozen-time analysis confining the initial propagation of temperature disturbances to a penetration distance is applicable to the predication of thermally induced instabilities in the system of moderately large Prandtl numbers.

CONCLUSIONS

The onset of thermal convection in a horizonal fluid layer heated from below with constant heat flux has been analyzed, using the modified frozen-time analysis. For this purpose a new mathematical approach by means of a rapidly converging power series has been developed.

The present analysis predicts the most reasonable stability conditions in the system of large Prandtl numbers in comparison with the existing experimental data. When $\tau_c\le 0.01$, the critical conditions are found to have the relations of $Ra_c = 14.1\ \tau_c^{-1}$ and $a_c = 0.248\ Ra_c^{1/4}$. Also, in this range it is shown that growth period of disturbances is inversely proportional to $Ra_c^{1/2}$. Considering all the present results, it is concluded that the extent of penetration of temperature disturbances in a fluid layer plays a dominant role in determining the critical conditions causing the onset of thermal convection.

The results presented here complement the existing results for a fluid layer undergoing a step change in temperature by Kihm, Choi and Yoo [15].

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NOMENCLATURE

- a: dimensionless wave number
- A_j: coefficients in equation (11)
- b_n: coefficients in equation (20)
- B_j: coefficients in equation (12)
- C_j: coefficients in equation (19)

d: fluid depth [m]
D: differential operator, $\frac{d}{d\zeta}$
$\underline{F}$: column matrix defined by equation (24)
g: acceleration of gravity [9.807 m /s^2]
$\underline{G}$: column matrix defined by equation (25)
k: thermal conductivity [W/m °K]
Nu: Nusselt number, θ_w^{-1}
Pr: Prandtl number, ν/α
q_w: lower surface heat flux [W/m^2]
R: Rayleigh number based on ΔT, $g\beta\Delta Td^3/\nu\alpha$
Ra: Rayleigh number based based on q_w, $g\beta q_w d^4/\alpha k\nu$
t: time [s]
T: temperature [°K]
T_i: initial or upper wall temperature [°K]
w: dimensionless vertical velocity, Wd/α
W: vertical velocity [m/s]
z: dimensionless vertical distance, Z/d
Z: vertical distance [m]

Greek symbols

α : thermal diffusivity [m^2/S]
β : thermal expansivity [°K^{-1}]
γ : coefficient in equation (21), aδ
δ : dimensionless thermal penetration distance
ΔT: surface-to-surface temperature difference, T_w-T_i
ζ : reduced vertical distance, z/δ
θ : dimensionless temperature, k(T-T_i)/q_wd
θ^*: amplitude function of temperature in equation (5)
ν : kinematic viscosity [m^2/s]
τ : dimensionless time, tα/d^2
ω^*: amplitude function of vertical velocity in equation (5)

Subscripts

A: region inside of penetration distance
B: region outside of penetration distance
C: critical state
O: base state
W: lower surface wall

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